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Modeling of shells with three-dimensional finite elements

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Abstract

Some aspects of three-dimensional analysis of shells are discussed, comparing 3d-shell formulations, surface oriented shell formulations and three-dimensional solid elements ("bricks"). Comparison is made with respect to theoretical formulation, finite element technology and consistency. Advantages and drawbacks of the different concepts are discussed, distinguishing the case of thin shells, where locking effects play a prominent role, and the analysis of three-dimensional structures ("very thick" shells). In this context a fundamental dilemma appears, namely the impossibility to design an element which is completely free of locking and passes the patch test at the same time.

1. Introduction

Finite element analysis of shells is a standard procedure in many engineering applications. Throughout the past fifteen years significant progress has been made in the field of three-dimensional shell theories and related finite elements (also called "solid shell" elements). The main benefits are approximate consideration of three-dimensional stress states, ease of implementation of three-dimensional constitutive laws and simplicity (no rotational degrees of freedom involved).

Sometimes application of shell finite elements is totally abandoned in favor of a discretization with three-dimensional solid finite elements. Standard 3d-solid finite elements, however, are often not suited to predict the behavior of shells properly. Locking effects may lead to significant errors for thin shells. Even well-established "locking-free" solid elements may fail in thin shell analysis. The problem can be avoided by transferring concepts of finite element technology from (3d) shell elements to solid elements.

A particularly interesting phenomenon in this context is *trapezoidal locking* (also called *curvature thickness locking* in the context of 3d-shell elements) and its relationship to passing the constant stress patch test. There are strong indications that obtaining a locking-free formulation and passing the patch test are mutually exclusive (a result already anticipated by Richard MacNeal in a discussion about distortion sensitivity of finite elements).

2. Three different archetypes of three-dimensional elements for shell analysis

Without going into technical details of mathematical formulations, three different philosophies for designing finite elements which are feasible for both thick and thin shell analysis, using unmodified three-dimensional constitutive laws are described in this section (see Figure 1 for an illustration).

3d-shell finite elements have become popular (particularly in Germany) at the beginning of the 90s, the works of Simo et al. [4] and Büchter et al. [4] being two of the decisive pioneering contributions. They rely on the classical concept of a mid-surface, equipped with displacement degrees of freedom, rotations (or difference displacements) as well as some higher order parameters, for instance describing the thickness change of the shell. A typical representative of this class is a so-called *7-parameter formulation*, utilizing three displacements of the mid-surface, three components of a difference vector (naturally including a constant thickness stretch) plus one additional strain (or displacement) parameter to realize a linear distribution of transverse normal strains. The latter is necessary to avoid *Poisson thickness locking* or, in other terms, to make the formulation asymptotically correct for bending.

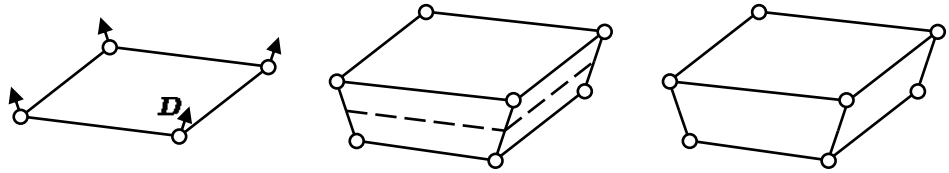

Figure 1: 3d-shell, surface oriented shell and “shell-like” 3d-solid

Unless the three-dimensional constitutive equations are somehow manipulated, all strain components need to be linear through the thickness in order to correctly model bending. In this spirit, quadratic terms are not needed for an asymptotically correct shell model and they are usually dropped from the formulation. Contribution of these higher order terms to the strain energy is negligible in most cases. This assumption is commonplace for practically all classical and three-dimensional shell models.

The mechanical ingredients of *surface oriented* shell formulations are identical to those of 3d-shells. The decisive difference is the usage of a three-dimensional parameterization of geometry and displacements with *absolute* values for position vectors and displacements rather than working with a director field $\boldsymbol{D}$ and difference displacements $\boldsymbol{w}$, see Figure 2. Surface oriented shell finite elements may be seen as hybrids of three-dimensional finite elements and shell elements. From the former they inherit their “outer appearance”, i.e. geometry and nodal degrees of freedom. And from the latter the “inner life” is deduced, particularly the use of stress *resultants* (membrane forces, bending moments etc.), rather than stresses and the aforementioned reduction strains to linear functions in thickness direction.

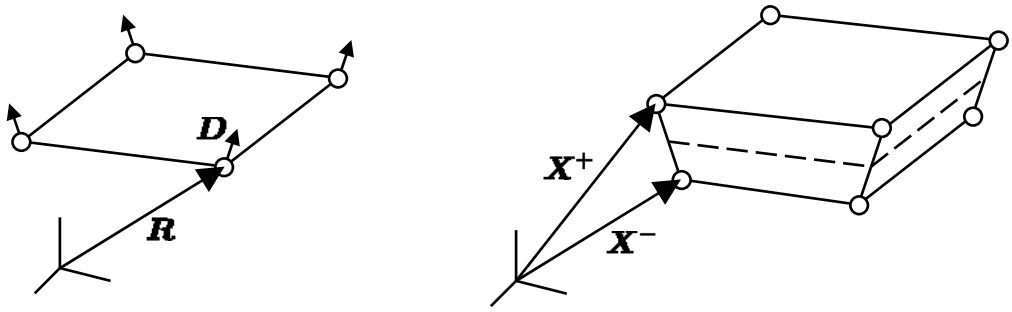

Figure 2: “classical” 3d-shell versus surface oriented 3d-shell

The concept of a surface-oriented shell model described in the previous section implies the question why not simply using 3d-elements for shell analysis. The nodal degrees of freedom are exactly the same and the 7th parameter is naturally included in many 3d-solid finite elements by certain means of element technology which are needed to avoid volumetric locking (EAS, Simo and Rifai [4]). The decisive differences of three-dimensional solid elements compared to three-dimensional (or surface oriented) shell elements are

1. that there are no distinct “in-plane” and “thickness” directions and
2. three-dimensional strains and stresses, rather than “resultants” are used, i.e. there are no separate expressions for membrane strains, curvatures, bending moments or transverse shear forces.

As a consequence, in a three-dimensional finite element the quadratic terms of the through-the-thickness strain distribution are not skipped – they *cannot* be skipped, because they do not appear separately. This is not a disadvantage because it does not affect computational cost. It may be even advantageous to have these terms included, as we will see later.

There is, however, a drawback in comparison to shell elements: From the point of view of element technology it is desirable to distinguish in-plane and out-of-plane strain and stress components, (e.g. when special formulations ought to be applied to avoid *transverse* shear locking or trapezoidal locking) as well as constant and linear ones (e.g. membrane and bending strains when membrane locking ought to be avoided). In fact, the crucial problem is trapezoidal locking (related to transverse normal strains) as will be demonstrated in the next section.

3. Trapezoidal locking and the patch test

We compare, in a numerical experiment, performance of state of the art solid elements and shell elements. A cylindrical shell with clamped boundaries is subject to uniform external pressure. A linear pre-buckling analysis is performed, based on solving the corresponding eigen value problem. Two different setups are considered: a relatively thick shell, with a radius-to-thickness ratio of 100 and a thin shell with slenderness 500. The commercial finite element package ANSYS is used as solver. The solid elements implemented in ANSYS use the enhanced assumed strain method, representing widely used, and state of the art brick elements. Comparison is made to a solution using conventional Kirchhoff-Love type shell elements which are free from locking for the problem at hand (similar results are obtained with well-formulated 3d-shell elements).

	mesh	shell elements	solid elements
thick shell	coarse	$\lambda_{crit.} = 1.02$	$\lambda_{crit.} = 1.7$
	fine	$\lambda_{crit.} = 1.0$ (reference)	$\lambda_{crit.} = 1.02$
thin shell	coarse	$\lambda_{crit.} = 1.02$	$\lambda_{crit.} = 13.0$
	fine	$\lambda_{crit.} = 1.0$ (reference)	$\lambda_{crit.} = 1.7$

Table 1: Critical load factors for thick and thin shell, fine and coarse meshes

The numerical results, normalized with respect to the fine mesh shell solution are summarized in Table 1. The coarse meshes use 4608 degrees of freedom (in both shell and solid discretizations) and the fine mesh involves 18816 d.o.f. It can be seen that the coarse mesh solution using shell elements is already satisfactory, while the coarse mesh solution with solid elements is unacceptable in both cases (thick and thin cylinder). Moreover, the absolute errors are much larger for the thin cylinder in the solid element solutions – a typical symptom of locking. Note, that even for the fine mesh the critical load is overestimated by 70 %. Figure 3 shows that the solid elements do not only overestimate the buckling load but also predict a wrong buckling pattern.

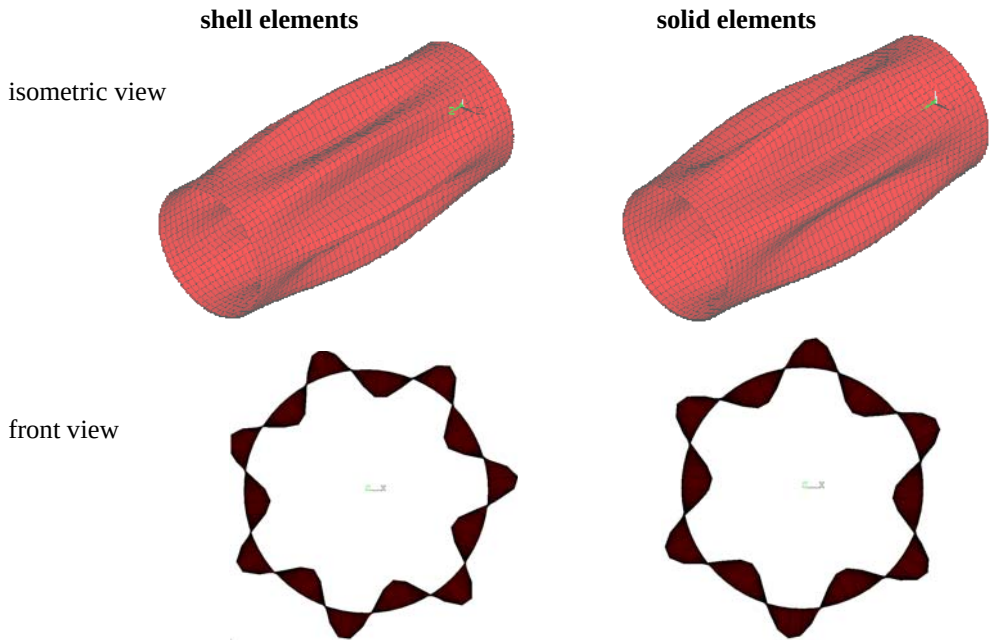

Figure 3: Comparison of buckling modes for shell elements and solid elements

Analyzing the problem setup and the involved element formulations one can identify trapezoidal locking as the reason for the observed behavior. Within a three-dimensional or surface oriented shell formulation it can be avoided following the idea of Betsch et al. [4] (see also Bischoff and Ramm [4]), modifying the constant part of

the transverse normal strains ε_{33} by an ANS formulation. As there is no "transverse" direction in a 3d-solid, transferring this concept is not straightforward. If one strives to keep element technology "isotropic", all normal strain components ought to be modified accordingly. This works fine for the numerical experiment documented in this section, but the resulting elements fail to pass the constant strain patch test. Strictly speaking, this means that these elements are not consistent and thus not convergent – a fact that is mostly considered unacceptable. The same holds for typical methods to avoid transverse shear locking.

We conclude that there is no way around an "anisotropic" element technology for 3d-solids, if these are expected to work as well as (3d) shell elements in the case of thin shells. This means that a certain thickness direction has to be nominated and the finite element formulation is adapted accordingly. If transverse shear strains and transverse normal strains are treated such that transverse shear locking and trapezoidal locking are avoided, 3d-solid elements may be used as efficiently as 3d-shell elements for thin shell analysis. These elements pass the patch test as long as the elements are distorted "in-plane", but still not in general three-dimensional situations. To be more precise: Constant strain states cannot be exactly represented if the thickness direction is not orthogonal to the other two directions.

The special treatment of transverse normal strains to avoid trapezoidal locking is one of the reasons for this problem. This observation is closely related to a discussion about distortion sensitivity of finite elements and passing the patch test, put forward by Richard MacNeal [4]. Removing these "tricks" from the element formulation leads to an element that passes the patch test (of course, trapezoidal locking re-enters the formulation).

For a surface oriented shell element the patch test is still not passed in this case. The reason is omitting the quadratic terms of the strain distribution trough the thickness. These are needed for exact representation of constant stress states for arbitrary meshes, because the metric is changing through the thickness.

One way out of this dilemma may be to accept the fact that the elements do not pass the patch test and favor formulations which are locking-free. Consistency, and thus convergence, may be achieved in a weaker sense by ensuring that those element distortions which are responsible for not passing the patch test vanish with mesh refinement. As these are related to deviation of the shell director from the shell normal, this seems to be a feasible approach.

3. Summary

Formulation of locking free 3d-solid elements for shells still has potential for improvement. Avoiding all locking effects and passing the patch test at the same time seem to be mutually exclusive. Standard "locking-free" 3d-solid elements usually suffer from trapezoidal locking and are therefore not suited for general shell analysis. The situation can be improved with an anisotropic element technology. Surface oriented shell elements use stress resultants and thus methods of element technology may be applied more purposeful.

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