CORRELATION COEFFICIENT AND COEFFICIENT OF VARIATION OF THE RATIO OF TWO VARIABLES

BU-158-M K. Choi October, 1963

ABSTRACT

This memorandom gives an asymptotic condition (in terms of the correlation coefficient) for the absolute value of the coefficient of variation of the ratio of two random variables to be less than or equal to the minimum of the absolute values of two coefficients of variation involved.

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CORRELATION COEFFICIENT AND COEFFICIENT OF VARIATION OF THE RATIO OF TWO VARIABLES[1]

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This memorandum gives an asymptotic condition (in terms of the correlation coefficient) for the absolute value of the coefficient of variation of the ratio of two random variables to be less than or equal to the minimum of the absolute values of two coefficients of variation involved.

Let X,Y be two random variables, ρ their correlation coefficient, $CV(\cdot)$ the coefficient of variation for the indicated variable. Assuming that the first two moments of $\frac{X}{Y}$ exist[2], the following results are immediate from the time-honored Taylor expansion (up to linear terms) of $\frac{X}{Y}$ around the value (EX, EY).

The condition on ρ for $|CV(\frac{X}{Y})| \leq$ minimun $\{|CV(X)|, |CV(Y)|\}$:

i) $EX > 0,\ EY > 0 \qquad \frac{1}{2\rho} \leq \frac{CV(X)}{CV(Y)} \leq 2\rho$

ii) $EX > 0,\ EY < 0 \qquad \frac{CV(X)}{CV(Y)} \leq \text{minimum}\ (2\rho, \frac{1}{2\rho})$

iii) $EX < 0,\ EY > 0 \qquad \frac{CV(X)}{CV(Y)} \geq \text{maximum}\ (2\rho, \frac{1}{2\rho})$

iv) $EX < 0,\ EY < 0 \qquad 2\rho \leq \frac{CV(X)}{CV(Y)} \leq \frac{1}{2\rho}$.

[1] This problem was raised by Dr. John Whitlock, Parasitology, Cornell University.

[2] This assumption precludes the case when X and Y are both normal variates.

When $\rho=0$, $|CV(\frac{X}{Y})| \geq \text{maximum}\ \{|CV(X)|, |CV(Y)|\}$.

Expanding $\frac{X}{Y}$ around (EX,EY), we obtain

$$E(\frac{X}{Y}) \doteq \frac{EX}{EY}$$

$$Var(\frac{X}{Y}) \doteq \frac{Var\ X}{(EY)^2} + \frac{(EX)^2\ Var\ Y}{(EY)^4} - 2\ Cov(XY)\ \frac{EX}{(EY)^2}$$

when the expansion is accurate to linear terms. Hence, the $[CV(\frac{X}{Y})]^2$ is asymptotically given by the following:

$$[CV(\frac{X}{Y})]^2 \doteq [CV(X)]^2 + [CV(Y)]^2 - 2\rho[CV(X)][CV(Y)] \quad .$$

It is immediate from the above relationship what conditions on ρ lead to the desired inequality, noting $CV(\cdot)$ is positive if and only if the mean is positive.