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A CURTAILED SEQUENTIAL PROCEDURE FOR SUBSET SELECTION OF MULTINOMIAL CELLS

by

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ABSTRACT

This paper deals with a curtailed sequential procedure for selecting a random size subset that contains the multinomial cell which has the largest cell probability. The proposed procedure R always selects the same subset as does the corresponding fixed-sample-size procedure, and thus achieves the same probability of a correct selection. But the sequential procedure accomplishes this with a smaller expected number of vector-observations than required by the fixed-sample-size procedure. Exact formulae for the savings are given as well as numerical calculations based on these formulae.

1. INTRODUCTION

This paper considers a closed sequential procedure for selecting a random size subset that contains the multinomial cell which has the largest cell probability. Multinomial selection problems have customarily been treated using two different approaches, namely, the indifference-zone approach and the subset approach. Bechhofer, Elmaghraby and Morse (1959), adopting the indifference-zone approach, proposed a fixed-sample-size procedure for selecting the multinomial cell which has the largest cell probability. Gupta and Nagel (1967), adopting the subset selection approach, proposed a fixed-sample-size procedure for selecting a random size subset that contains the multinomial cell which has the largest cell probability. Inverse sampling sequential procedures for these two approaches were studied by Cacoullos and Sobel (1966) and Panchapakesan (1971), respectively. Other sequential procedures which employ the indifference-zone approach for multinomial selection problems were given by Bechhofer, Kiefer and Sobel (1968), Ramey and Alam (1979) (see also Bechhofer and Goldsman (1985a)) and Bechhofer and Goldsman (1985b, 1986).

Bechhofer and Kulkarni (1984) considered a closed sequential procedure in which curtailment was applied to a generalized version of the selection goal of Bechhofer, Elmaghraby and Morse (1959). Both procedures were proved to achieve the same probability of a correct selection, uniformly in the unknown cell probabilities $\underset{\sim}{p} = (p_1, p_2, \ldots, p_k)$. The procedure of Bechhofer and Kulkarni always requires a smaller expected number of vector-observations to terminate sampling than required by the corresponding fixed-sample-size procedure of Bechhofer, Elmaghraby and Morse. Bechhofer and Goldsman (1985b, 1986) studied the performance of truncated versions of an open sequential sampling procedure proposed by Bechhofer, Kiefer and Sobel (1968) which also employ the indifference-zone approach. Thus far no article has been published dealing with a closed sequential procedure for the random subset selection approach. Motivated by the use of curtailment by Bechhofer and Kulkarni, we consider a closed curtailed sequential version (R) of the Gupta and Nagel (1967) fixed-sample-size procedure. We show that R always selects the same subset as does the Gupta-Nagel procedure, and hence achieves the same probability of a correct selection. But R accomplishes this with a smaller expected number of vector-observations than required by Gupta-Nagel.

The procedure R is defined formally in Section 2. In Section 3, we investigate some properties of the probability of a correct solution $(P\{CS|R\})$ and the expected size of the selected subset $(E\{S|R\})$. In Section 4, we give the formulae for the expected number of vector-observations $(E\{N|R\})$ to terminate sampling for $k = 2$ and 3 cells. These formulae can be generalized to larger k using a similar method. In Section 5, we provide a table of $E\{N|R\}$ for the so-called slippage configuration of the cell probabilities , and explain how it was computed.

2. THE SELECTION GOAL AND THE PROPOSED PROCEDURE

A multinomial distribution with k cells $\pi_1, \pi_2, \ldots, \pi_k$ is given; let the ordered values of the unknown cell probabilities $p_i \geq 0$ $(1 \leq i \leq k)$ with $\sum_{i=1}^{k} p_i = 1$ be denoted by $p_{[1]} \leq p_{[2]} \leq \ldots \leq p_{[k]}$, and the corresponding cells be denoted by $\pi_{(1)}, \pi_{(2)}, \ldots, \pi_{(k)}$. It is assumed that the values of the p_i and $p_{[j]}$ $(1 \leq i, j \leq k)$ are unknown, and the pairings of the π_i with the $p_{[j]}$ are completely unknown. The goal of the experimenter is to select a random size subset containing the cell $\pi_{(k)}$. A correct selection (CS) is defined as the selection of any subset of the k cells which contains the cell $\pi_{(k)}$. If more than one cell has a p-value equal to $p_{[k]}$, then one of the cells with the largest value is considered "tagged," and the selection is correct if this "tagged" cell is in the selected subset. Let P^* with $1/k < P^* < 1$ denote a specified constant. We require a procedure R which guarantees that for all $\underset{\sim}{p} = (p_1, p_2, \ldots, p_k)$ we have

$$P(CS|R) \geq P^*. \tag{2.1}$$

The procedure R takes vector-observations one-at-a-time until a certain stopping requirement is satisfied. Let n denote the largest number of vector-observations that the experimenters will be allowed to take. (This n is the same as the N of Gupta-Nagel; we have changed notation since our N

of Section 4 is a random variable.) The value of n may have been based on economic considerations. By stage m $(m \le n)$, we shall mean that a total of m vector-observations have already been taken. Let the random variable $Z_{i,m}$ $(1 \le i \le k,\ 1 \le m \le n)$ denote the frequency in cell π_i through stage m, and let D be a predetermined non-negative integer. For given (k, n, D) we now state the curtailed procedure (R) which guarantees the probability requirement (2.1).

Procedure R:

Sampling Rule: Take vector-observations one-at-a-time.

Stopping Rule: Stop sampling at the first stage m at which there exists a cell π_i such that

$$z_{i,m} > z_{j,m} + n - m + D \quad \text{for all} \quad j \ne i \ (i,j = 1,2, \ldots, k). \tag{2.2}$$

Selection Rule: Having stopped, include in the selected subset the cell π_i with observed frequency $z_{i,m}$ if and only if

$$z_{i,m} \ge z_{\max,m} - D \tag{2.3}$$

where $z_{\max,m} = \max(z_{1,m}, z_{2,m}, \ldots, z_{k,m})$.

Note: For given (k, P^*, n) values of D are tabled by Gupta and Nagel so that their fixed-sample-size procedure will guarantee (2.1).

It is clear from the above definition of Procedure R, that only the cell with the maximum frequency is selected when the sampling is stopped at stage $m < n$. When the sampling is stopped at stage n, the selection rule

(2.3) will be used to select a subset that contains one or more cells.

3. AN IMPORTANT PROPERTY OF PROCEDURE R

Procedure R possesses an important property relative to the corresponding fixed-sample-size procedure R_{GN} of Gupta and Nagel. Using the same notation as we used in defining procedure R in Section 2, we now state the procedure R_{GN}:

Procedure R_{GN}: A total of n vector-observations is taken in a single stage. Include in the selected subset the cell π_i with the observed frequency $z_{i,n}$ if and only if

$$z_{i,n} \geq z_{max,n} - D \tag{3.1}$$

where $z_{max,n}$ and D are defined as in (2.3).

We now state and prove the following result concerning the relation between the performance characteristics of procedures R and R_{GN}.

Theorem 3.1: For given (k,n) both R and R_{GN} select the same subset of the k cells if both use the same D. This result is uniform in $(p_1, p_2, \ldots, p_k)$.

Proof. The selection rule (2.3) is identical to (3.1) when $m = n$. Thus the same decision will be made by R and R_{GN} with any sampling outcomes that takes a total of n observations. Hence we need only consider the situation when the sampling with R is stopped at stage $m < n$. When this happens, we have $n - m > 0$. Let $z'_{i,m}$ denote the frequency associated with the cell which at termination has the largest frequency.

$$z_{j,m}+n-m < z'_{i,m} - D \quad \text{for all} \quad j \neq i \ (i,j = 1,2, \ldots, k) \tag{3.2}$$

Here $(n-m)$ is the maximum total number of vector-observations that can be taken to complete the experiment. Thus even if the sampling was terminated before a total of n observations was taken, the same decision of selecting the same one cell π_i will be made since

$$z_{j,n} \le z_{j,m} + (n-m) < z'_{i,m} - D \le z_{i,n} - D \quad \text{for all} \quad j \neq i. \tag{3.3}$$

Thus the same decision will be made using R or R_{GN}. As a consequence, $P(CS|R) = P(CS|R_{GN})$ and $E(S|R) = E(S|R_{GN})$ uniformly in $\underset{\sim}{p} = (p_1, p_2, \ldots, p_k)$.

<u>Remark 3.1</u>: The result of the above theorem is analogous to the corresponding result obtained by Bechhofer and Kulkarni (1984) concerning multinomial selection problems using the indifference-zone approach. However, our proof is much simpler than theirs since we use a strict inequality (>) in (2.2). The strict inequality in (2.2) leads to the <u>same</u> decision for both procedures R and R_{GN}. Theorem 3.1 is not true if a weak inequality ($\ge$) is used in (2.2) to replace the strict inequality.

<u>Remark 3.2</u>: As a consequence of Theorem 3.1, the configuration of the $p_i (1 \le i \le k)$ that minimizes $P(CS)$ is the same for both procedures R and R_{GN},

uniformly in n and k. It was shown in Gupta and Nagel (1967) that this configuration is of the form

$$(0, 0, \ldots, 0, s, p, \ldots p),\ s \le p \tag{3.4}$$

Thus the global minimum can be found by solving

$$\min_{\underline{p}} P(CS|k, n, D; \underline{p}) = \min_{2\le r\le k} \left[\min_{\frac{1}{r}\le p\le\frac{1}{r-1}} P(CS|(0, \ldots, 0, s, p, \ldots, p))\right]$$

where r is the number of positive p's in (3.4) and $s = 1-(r-1)p$. As pointed out in Gupta and Nagel, the minimum usually takes place at one end of the p-interval in question, i.e., for $p = 1/r$ or for $p = 1/(r-1)$. When we used our computing program to prepare our Table I of $E(N|R)$ for $k = 2(1)8$ and $n = 5(1)20$, we found that the only case for which the minimum was attained in the interior of the p-interval occurs when $k = 3$, $n = 6$ and $D = 4$, which is the same case as was noted by Gupta and Nagel.

4. FORMULAE FOR $E(N|R)$: $k = 2$ AND 3

Let N denote the random number of vector-observations to terminate sampling using R. We now derive formulae for $E(N|R)$ for the cases $k = 2$, $D = 0$ and 1 and a general formula for $k = 3$.

For $k = 2,\ D = 0$:

Let $B(x|n,p) = \binom{n}{x} p^x (1 - p)^{n-x}$ denote the binomial probability.

When n is odd,

$$E(N|R) = \sum_{i=\frac{n+1}{2}}^{n} i \cdot [p_1\, B(\tfrac{n-1}{2}|i - 1, p_1) + p_2 B(\tfrac{n-1}{2}|i - 1, p_2)]. \tag{4.1a}$$

When n is even,

$$E(N|R) = \sum_{i=\frac{n}{2}+1}^{n} i[P_1 B(\tfrac{n}{2}|i-1, p_1) + p_2 B(\tfrac{n}{2}|i-1, p_2)] + nB(\tfrac{n}{2}|n, p_1). \quad (4.1b)$$

<u>For $k = 2$, $D = 1$</u>:

When n is odd,

$$E(N|R) = \sum_{i=\frac{n+1}{2}+1}^{n} i[p_1\, B(\tfrac{n+1}{2}|i-1, p_1) + p_2 B(\tfrac{n+1}{2}|i-1, p_2)]$$

$$+ n[B(\tfrac{n+1}{2}|n, p_1) + B(\tfrac{n+1}{2}|n, p_2)]. \quad (4.2a)$$

When n is even,

$$E(N|R) = \sum_{i=\frac{n}{2}+1}^{n} i[p_1\, B(\tfrac{n}{2}|i-1, p_1) + p_2\, B(\tfrac{n}{2}|i-1, p_2)]$$

$$+ nB(\tfrac{n}{2}|n, p_1). \quad (4.2b)$$

For $k = 3$ or larger, we do not have a simple formula for $E(N|R)$ similar to the ones in (4.1) and (4.2) for the case $k = 2$. However, $E(N|R)$ for $k = 3$ can be expressed in the following manner:

$$E(N|R,\ k = 3) = \sum^{*}_{(x_1, x_2, x_3)} x\, c_X(x_1, x_2, x_3)\, p_1^{x_1}\, p_2^{x_2}\, p_3^{x_3}. \quad (4.3)$$

The expression is quite easy to evaluate using a computing algorithm, and can be generalized to any k without any difficulty. Here Σ^{*} is taken over all (x_1, x_2, x_3) such that $\sum_{i=1}^{3} x_i = x \le n$. The coefficient $c_X(x_1, x_2, x_3)$ is a function of non-negative integers x_1, x_2, and x_3 defined as follows:

$$c_X(x_1, x_2, x_3) = G_X(x_1, x_2, x_3) - G_{X-1}(x_1 - 1, x_2, x_3) - G_{X-1}(x_1, x_2 - 1, x_3)$$

$$- G_{X-1}(x_1, x_2, x_3 - 1)$$

where

$$G_X(x_1,x_2,x_3) = \begin{cases} \dfrac{x!}{x_1!x_2!x_3!} & \text{if } x = n \text{ and } x_{[3]} \geq n - x_{[2]} + D \\ & \underline{\text{or}}\ x < n \text{ and } x_{[3]} > n - x_{[2]} + D \\ 0 & \text{otherwise.} \end{cases} \quad (4.4)$$

Here $x_{[3]} \geq x_{[2]} \geq x_{[1]}$ are the ordered values of the observed frequencies x_1, x_2, and x_3.

Examples: Let $\binom{\alpha}{\beta}$ denote the binomial coefficient, and $\binom{\alpha}{\alpha_1\ \alpha_2 \ldots \alpha_i}$ denote the multinomial coefficient. For $n = 5$, $D = 0$, $k = 3$, we have

$$\begin{aligned} E(N|R) &= 3\tbinom{2}{0}\ (p_1{}^3 + p_2{}^3 + p_3{}^3) \\ &+ 4\tbinom{3}{1}\ (p_3{}^3p_1 + p_2{}^3p_1 + p_1{}^3p_2 + p_3{}^3p_2 + p_2{}^3p_3 + p_1{}^3p_3) \\ &+ 5\tbinom{4}{211}\ (p_3{}^3p_1p_2 + p_2{}^3p_1p_3 + p_1{}^3p_2p_3) \\ &+ 5\tbinom{4}{22}\ (p_3{}^3p_2{}^2 + p_3{}^3p_1{}^2 + p_2{}^3p_3{}^2 + p_2{}^3p_1{}^2 + p_1{}^3p_2{}^2 + p_1{}^3p_3{}^2) \\ &+ 5\tbinom{5}{221}\ (p_3{}^2p_2{}^2p_1 + p_3{}^2p_2p_1{}^2 + p_2{}^2p_1{}^2p_3). \end{aligned}$$

For $n = 5$, $D = 1$, $k = 3$, we have

$$\begin{aligned} E(N|R) &= 4\tbinom{3}{0}\ (p_1{}^4 + p_2{}^4 + p_3{}^4) \\ &+ 5\tbinom{4}{1}\ (p_3{}^4p_1 + p_3{}^4p_2 + p_2{}^4p_1 + p_2{}^4p_3 + p_1{}^4p_2 + p_1{}^4p_3) \\ &+ 5\tbinom{5}{311}\ (p_3{}^4p_1p_2 + p_1{}^4p_3p_2 + p_2{}^4p_1p_3) \\ &+ 5\tbinom{5}{2}\ (p_3{}^3p_2{}^2 + p_3{}^3p_1{}^2 + p_2{}^3p_1{}^2 + p_2{}^3p_3{}^2 + p_1{}^3p_3{}^2 + p_1{}^3p_2{}^2) \\ &+ 5\tbinom{5}{221}\ (p_3{}^2p_2{}^2p_1 + p_3{}^2p_2p_1{}^2 + p_2{}^2p_1{}^2p_3). \end{aligned}$$

5. TABLES AND REMARKS

To compare the expected number of vector-observations of procedure R relative to that of n of the competing procedure R_{GN}, we present in Table 1 the numerical values of $E(N|R)$ under the so called slippage configuration $(p, p, \ldots, Ap)$ for $k = 2(1)8$; $n = 5(1)15$; $D = 0,1,2$; and $A = 1,3,5$. From Theorem 3.1, $P(CS|R)$ and $E(S|R)$ are the same for both R and R_{GN}. These later quantities are tabled in Gupta-Nagel for the same (k, n, A, D)-values. Based on the table, we can draw the following conclusions concerning the saving $n - E(N)$ in the expected sample size:

Remark 5.1: For fixed (n, k, D), the saving increases with increasing A. When A approches ∞, the configuration $\underset{\sim}{p}$ approaches $(0, 0, 0, \ldots, 1)$. The expected sample size for any (n, k, D) is then

$$E\{N|\underset{\sim}{p} = (0, 0, 0, \ldots, 1)\} = \left[\frac{n+D}{2} \right]^{+} \tag{5.1}$$

where $[a]^{+}$ is the smallest integer greater than a. The saving in this case is

$$n - E\{N|\underset{\sim}{p} = (0, 0, 0, \ldots, 1)\} = \left[\frac{n-D-1}{2} \right]^{+}. \tag{5.2}$$

Remark 5.2: For fixed (n, A, D), the saving decreases with increasing k. For fixed (k, A, D), the saving increases with increasing n for n either odd or even.

Remark 5.3: Panchapakesan (1971) does not calculate $E(N)$-values for his procedure although he does give a formula for $E(N)$. He points out that he does not provide a theoretical or numerical comparison of n for the

Gupta-Nagel procedure and $E(N)$ for his procedure because it is difficult to equate the $P(CS)$ for the two procedures. The same is, of course, true for our procedure since ours achieves the same $P(CS)$ as does the Gupta-Nagel procedure. We conjecture that if the $P(CS)$ for the Panchapakesan procedure and our procedure were equated, ours would have the smaller $E(N)$, at least for k moderately large and P^* and A close to unity.

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TABLE I

THE EXPECTED SAMPLE SIZE E(N) FOR PROCEDURE R

A = 1, D = 0

n \ k	2	3	4	5	6	7	8
5	4.125	4.555	4.734	4.824	4.875	4.907	4.928
6	5.500	5.580	5.688	5.766	5.821	5.859	5.887
9	7.539	8.342	8.535	8.678	8.752	8.790	8.815
10	9.047	9.309	9.501	9.621	9.714	9.772	9.808
13	11.067	12.110	12.358	12.531	12.616	12.676	12.725
14	12.648	13.009	13.327	13.493	13.597	13.658	13.703
17	14.662	15.843	16.197	16.396	16.510	16.598	16.667
18	16.291	16.805	17.167	17.369	17.486	17.573	17.642

A = 1, D = 1

n \ k	2	3	4	5	6	7	8
5	4.875	4.963	4.934	4.992	4.995	4.997	4.998
6	5.500	5.827	5.922	5.958	5.975	5.984	5.989
9	8.555	8.705	8.864	8.911	8.933	8.948	8.959
10	9.047	9.674	9.812	9.893	9.928	9.945	9.956
13	12.229	12.539	12.746	12.828	12.876	12.909	12.933
14	12.648	13.515	13.702	13.814	13.864	13.896	13.921
17	15.920	16.381	16.631	16.749	16.822	16.869	16.898
18	16.290	17.311	17.592	17.731	17.805	17.857	17.891

A = 1, D = 2

n \ k	2	3	4	5	6	7	8
5	4.875	4.963	4.984	4.992	4.995	4.997	4.998
6	5.938	5.988	5.996	5.998	5.999	6.000	6.000
9	8.555	8.916	8.956	8.974	8.984	8.990	8.993
10	9.734	9.864	9.952	9.973	9.982	9.988	9.992
13	12.229	12.781	12.903	12.944	12.967	12.980	12.987
14	13.496	13.759	13.892	12.937	13.960	13.974	13.982
17	15.920	16.670	16.839	16.906	16.944	16.962	16.973
18	17.251	17.653	17.826	17.896	17.935	17.958	17.972

A = 3, D = 0

n \ k	2	3	4	5	6	7	8
5	3.773	4.238	4.500	4.654	4.750	4.813	4.856
6	5.031	5.306	5.475	5.596	5.684	5.749	5.798
9	6.540	7.534	7.961	8.234	8.412	8.530	8.610
10	7.848	8.392	8.830	9.110	9.308	9.450	9.551
13	9.265	10.646	11.276	11.693	11.971	12.166	12.312
14	10.588	11.420	12.101	12.545	12.855	13.073	13.232
17	11.963	13.667	14.510	15.080	15.474	15.762	15.978
18	13.292	14.421	15.319	15.918	16.343	16.652	16.885

A = 3, D = 1

n \ k	2	3	4	5	6	7	8
5	4.680	4.867	4.935	4.965	4.979	4.987	4.991
6	5.031	5.517	5.738	5.846	5.904	5.937	5.957
9	7.693	8.123	8.459	8.643	8.747	8.813	8.857
10	7.848	8.886	9.293	9.535	9.679	9.767	9.824
13	10.514	11.280	11.859	12.205	12.426	12.574	12.678
14	10.588	12.040	12.679	13.081	13.333	13.501	13.619
17	13.255	14.337	15.158	15.674	16.021	16.261	16.430
18	13.292	15.086	15.961	16.530	16.908	17.171	17.358

A = 3, D = 2

n \ k	2	3	4	5	6	7	8
5	4.680	4.876	4.935	4.965	4.979	4.987	4.991
6	5.762	5.922	5.968	5.985	5.992	5.996	5.998
9	7.693	8.476	8.725	8.836	8.896	8.932	8.954
10	8.897	9.336	9.645	9.791	9.867	9.910	9.937
13	10.514	11.775	12.281	12.550	12.707	12.804	12.865
14	11.781	12.593	13.153	13.461	13.642	13.755	13.829
17	13.255	14.934	15.687	16.135	16.413	16.589	16.704
18	14.555	15.708	16.527	17.015	17.323	17.524	17.656

A = 5, D = 0

n \ k	2	3	4	5	6	7	8
5	3.532	3.932	4.210	4.403	4.539	4.637	4.709
6	4.710	5.017	5.213	5.358	5.471	5.560	5.631
9	5.980	6.796	7.284	7.623	7.872	8.059	8.202
10	7.176	7.671	8.098	8.429	8.689	8.896	9.060
13	8.394	9.509	10.162	10.643	11.013	11.305	11.541
14	9.593	10.268	10.900	11.394	11.788	12.105	12.362
17	10.798	12.157	12.958	13.574	14.062	14.460	14.788
18	11.998	12.856	13.667	14.302	14.817	15.239	15.591

A = 5, D = 1

n \ k	2	3	4	5	6	7	8
5	4.517	4.739	4.847	4.904	4.937	4.957	4.969
6	4.710	5.177	5.462	5.637	5.747	5.819	5.867
9	5.980	6.796	7.284	7.623	7.872	8.059	8.202
10	7.176	8.096	8.607	8.950	9.192	9.366	9.493
13	9.581	10.239	10.834	11.281	11.621	11.882	12.086
14	9.593	10.816	11.513	12.022	12.405	12.699	12.929
17	11.995	12.846	13.640	14.249	14.730	15.114	15.422
18	11.998	13.460	14.318	14.977	15.493	15.905	16.240

A = 5, D = 2

n \ k	2	3	4	5	6	7	8
5	4.517	4.739	4.847	4.904	4.937	4.957	4.970
6	5.598	5.814	5.904	5.947	5.969	5.980	5.987
9	7.130	7.918	8.314	8.535	8.672	8.762	8.824
10	8.294	8.772	9.154	9.409	9.574	9.685	9.762
13	9.581	10.748	11.381	11.807	12.106	12.322	12.481
14	10.771	11.495	12.136	12.596	12.931	13.176	13.360
17	11.995	13.437	14.261	14.871	15.328	15.677	15.945
18	13.192	14.126	14.979	15.619	16.112	16.493	16.790

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